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Technical Notes

Propeller Wall-Blockage Performance Corrections

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Nomenclature

A = flow cross-sectional area
C_T = thrust coefficient
P = power
p = pressure
T = thrust
V = velocity
v = nondimensional velocity
V_P = power-based velocity
α = blockage coefficient
ρ = fluid density

Subscripts

a = ambient freestream conditions in wind tunnel
c = flow conditions inside flow domain/conduit at downstream infinity
f = conditions in unencumbered, free-air state
o = propeller wake flow properties at downstream outlet
p = conditions at propeller plane
1, 2 = properties fore and aft of propeller/blade, respectively

Introduction

STUDIES of wall-blockage effects on propellers have continued for nearly 100 years with no method yet presented for directly correcting measured or calculated thrust performance. Many, in fact, nearly all efforts to date employ Glauert's [1] freestream velocity adjustments (see also [2]), apparently first introduced by Wood and Harris [3]. Glauert [1] employed a momentum-balance/actuator-disc model to provide numerical estimates and an approximation formula for adjusting the open, unencumbered free-air flow speed so as to generate the same thrust on the propeller as one would measure in the wind tunnel at a set velocity. The resulting velocity adjustment relation provided has subsequently been used throughout the propulsion industry. Assessments of the method's utility have been conducted by Loeffler and Steinhoff using inviscid modeling for a shrouded propeller [4], Mikkelsen and Sørensen using computational fluid dynamics (CFD) analysis for wind turbines [5], and Fitzgerald using propeller wind-tunnel experiments [6]. For propellers, the results are found to be limited in scope and seemingly anomalous, thus prompting a need for further analysis seeking clarification of this important issue.

The current effort extends the author's recent analysis, presented in [7], for wind turbine wall-blockage corrections to propellers. It is based on the formulation provided by Glauert [1] and provides, for the first time, exact and approximate solutions for directly correcting propeller thrust performance for wall blockage. Additionally, it exposes a heretofore understood but unarticulated inherent limit to testing or numerically modeling propellers in conduits. Most important, it shows that, consistent with the results observed in [4,6], the needed blockage corrections to thrust levels are virtually nil for up to 25% of area blockage by the propeller.

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Governing Equations

Following Glauert [1] and Mikkelsen and Sørensen [5], a momentum-balance/actuator-disc model of a propeller/turbine blade is employed for the control volume shown in Fig. 1. Momentum, mass, and energy balances are applied to the flow structure, and the cuts are shown with the following assumptions applied: 1) inviscid incompressible flow through a conduit of constant cross-sectional area A_c, which need not be axisymmetric; 2) flow with specified uniform pressure p_a and velocity V_a entering at upstream infinity; 3) flow exiting at downstream infinity at uniform pressure p_o with a slipstream between the mainstream fluid exiting at velocity V_c and the fluid exiting at velocity V_o that passed through the actuator-disc model of the propeller (the three variables, p_o, V_c, and V_o, are treated as unknowns); and 4) the cuts around the propeller shrink to the surface.

To address both the static case, $V_a = 0$, as well as forward flight cases, $V_a > 0$, within a single formulation, the nondimensionalization scheme of [8] is adopted to write the governing equations as

$$(2v_o + v_c - v_a) = 2v_o(v_o + v_c)(v_o^2 - v_c^2) \tag{1}$$

$$2v_o(v_a - v_c)(v_o + v_c) = \alpha \tag{2}$$

where the nondimensional velocities are defined as

$$v_o \equiv V_o/V_P, \qquad v_c \equiv V_c/V_P, \qquad v_a \equiv V_a/V_P \tag{3}$$

with

$$V_P \equiv (4P/\rho A_p)^{1/3} \tag{4}$$

In Eq. (4), P is the power put into the flow by the propeller, and the flow domain/conduit area ratio or blockage coefficient has been defined as

$$\alpha \equiv A_p/A_c \tag{5}$$

The thrust coefficient can be written as

$$C_T \equiv \frac{T}{(1/2)\rho A_p V_P^2} = (v_o^2 - v_c^2) \tag{6}$$

The nondimensionalization scheme, used previously in Eqs. (1–6), is different than traditionally employed throughout the propulsion field, such as discussed in [1–5]. However, it is the one dictated by the governing equations themselves, reducing them to the simplicity of Eqs. (1) and (2) containing the two parameters α and v_a. The power-based velocity term V_P of Eq. (4) can be easily shown to be linearly proportional to the propeller's revolutions per minute and diameter; thus, v_a of Eq. (3) can be viewed as a first-principles-based more general representation of the traditional propeller advance ratio. Similarly, the thrust coefficient C_T of Eq. (6) can be shown to be a generalization of the traditional thrust parameter employing the fourth power of the rotor diameter and square of the rotor's revolutions per minute.

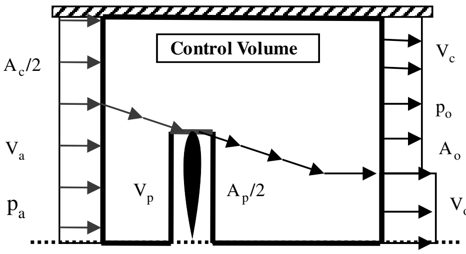

Fig. 1 Shroud control volume.

Glauert [1], apparently following the suggestion of Wood and Harris [3], solved the governing equations, not for the thrust but rather for the adjustment needed to the forward velocity in free air V_{af} to obtain the same thrust as that measured in a wind tunnel at velocity V_a at the same power setting. Following Glauert [1] or Mikkelsen and Sørensen [5], in the current nomenclature, this can be written as

$$V_{af}/V_a = \frac{v_p}{v_a} - \frac{1}{4}\frac{C_T}{v_a v_p} \tag{7}$$

where it can be shown that the velocity at the propeller station is

$$v_p \equiv \frac{V_p}{V_a} = \frac{v_o(v_a - v_c)}{\alpha(v_o - v_c)} \tag{8}$$

In the current study, instead of the velocity correction factor given by Eq. (7), attention will be given to the thrust correction factor; that is, $T_f/T = C_{Tf}/C_T$, where T_f and C_{Tf} are the thrust and thrust coefficients, respectively, to be expected for the identical configuration tested in free air at the same power setting. In this case, $\alpha = 0$ in Eqs. (1) and (2), which leads to $v_c = v_a$ and a cubic equation for v_{of}, for which the exact solution was given in [8] as

$$v_{of} = (1/2)^{1/3}[1 + 16/27v_a^3 + \sqrt{1 - 32/27v_a^3}]^{1/3} + (1/2)^{1/3}[1 + 16/27v_a^3 - \sqrt{1 - 32/27v_a^3}]^{1/3} - v_a/3 \tag{9}$$

The thrust coefficient is then given by Eq. (6) as

$$C_{Tf} = v_{of}^2 - v_a^2 \tag{10}$$

so that the thrust correction factor is

$$T_f/T = C_{Tf}/C_T = (v_{of}^2 - v_a^2)/(v_o^2 - v_a^2) \tag{11}$$

To the author's knowledge, values of T_f/T from Eq. (11) have never before been provided, even though they are more relevant to the blockage prediction issue and are no more difficult to obtain than the velocity correction factors of Eq. (7).

Before proceeding to analyze the results obtained from solving the preceding equation set, it is first useful to expose a somewhat obvious, well understood, but as yet unarticulated, embedded limitation encountered when v_a decreases toward zero to the point that $v_c = 0$. This occurs when the power input is so high (i.e., v_a is low) that all the flow entering the conduit of Fig. 1 from the left is drawn through the propeller. Any further increase in power (or further reduction in the freestream velocity) would demand flow entering the conduit from the downstream outlet: i.e., $v_c < 0$. To avoid this condition, physical and/or numerical experiments must limit the power applied through the propeller, depending on the freestream velocity and the model blockage level. This limit condition is easily recovered from Eqs. (1) and (2) to write the minimum value of v_a to be

$$v_{a\,\min} \equiv \alpha/[4(1 - \alpha/2 + \sqrt{1-\alpha})]^{1/3} \tag{12a}$$

which for small values of α becomes

$$v_{a\,\min} \approx \alpha/2 \tag{12b}$$

Using this value in Eqs. (6–11) with $v_c = 0$ then provides the values of the thrust and velocity correction factors at this limit condition.

As a final point, approximations are developed for the velocity and thrust correction factors for small α in Eqs. (1–11) to give

$$V_f/V_a \approx 1 - \frac{\alpha}{4}\frac{C_T}{v_a\sqrt{C_T + v_a^2}} \tag{13a}$$

and

$$T_f/T = C_{Tf}/C_T \approx 1 - \frac{\alpha}{2}\frac{C_T}{(3C_T^2 + 2v_a)\sqrt{C_T + v_a^2}} \tag{13b}$$

both of which are only valid for velocity ratios equal to or greater than $v_{a\,\min}$, defined by Eq. (12). Equation (13a) for the velocity correction factor is identical to that derived and presented by Durand in [2], recast here into the current variables. Equation (13b) for the thrust correction factor is entirely new.

Results and Discussion

Figures 2 and 3 provide the exact values of the wake and outlet velocity components, v_o and v_c, for blockage factors α varying from zero to one. To obtain these values, Eqs. (1) and (2) were solved iteratively for given values of α and v_a. Also shown are the limit value loci and limit values for each α. Note that, in Fig. 2, the dip to a minimum value for the wake velocity values occurs because the relative power input is decreasing as the freestream velocity ration v_a increases. Eventually, as v_a continues to increase (or relative power decreases) the wake velocity v_o must asymptotically approach the freestream value v_a for all values of blockage. The case of $\alpha = 1$ is the extreme where the propeller fills the flow conduit; thus, $v_o = v_a$ for all v_a above $v_{a\,\min}$. If v_a decreases further (or input power increases further), the system stalls because it is trying to pass more flow than the inlet is delivering. Figure 3 shows that, as expected, the tunnel outlet velocity v_c equals the freestream value for $\alpha = 0$ and decreases below the freestream level for all $\alpha > 0$ until it reaches zero.

The values of v_o and v_c, shown in Figs. 2 and 3, were employed in Eqs. (6–8) to determine the velocity correction factor, V_{af}/V_a, shown in Fig. 4. Also shown is the limit curve for $v_c = 0$, which is easily determined using Eq. (12a) in Eqs. (1–8) plus the correction factor values provided by Glauert [1] for the $\alpha = 0.25$ case. These precisely match the current values for the range they were provided. Figure 4 also provides Glauert's approximate values from Eq. (13b), showing remarkable accuracy up to blockage levels of 0.5. Most notable in Fig. 4 is the precipitous drop in the correction factor toward an analytically verifiable lower limit value of 0.5 as v_a decreases, either due to lower freestream velocity or higher power input. This behavior is present for all values of α as manifest, rather dramatically, by the $\alpha = 0.01$ case of Fig. 4.

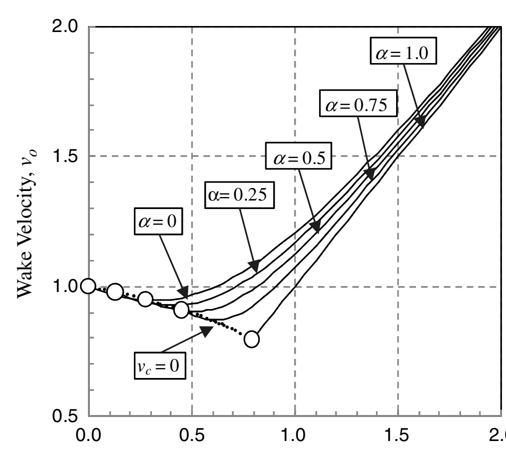

Fig. 2 Wake velocity.

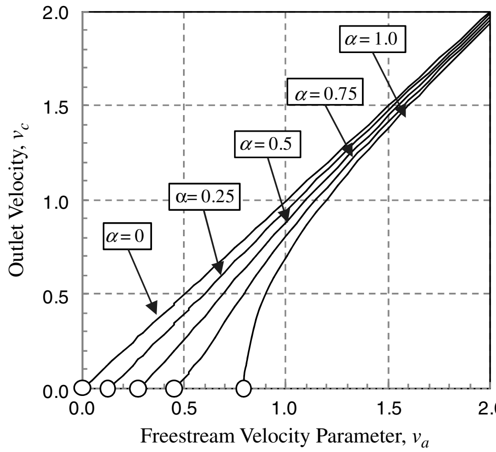

Fig. 3 Outlet velocity.

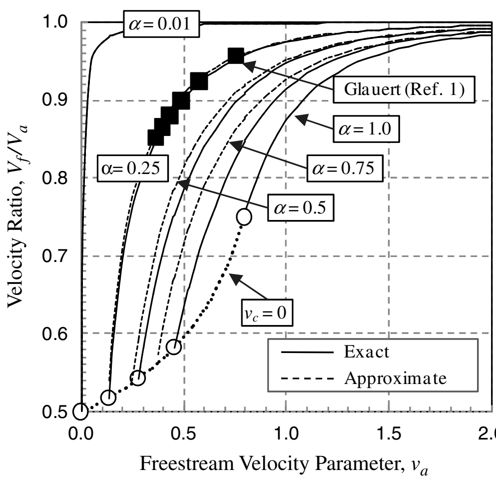

Fig. 4 Velocity correction factor.

Figure 5 presents the thrust correction factor, T_f/T, for the same conditions as Fig. 4, again along with the limit curves for $v_c = 0$ and the approximate values given by Eq. (13b). Again, the approximate values show surprisingly reasonable accuracy up to blockage levels of 0.5. More important, the thrust corrections are seen to be very small for all but the highest values of α. For example, for $\alpha = 0.25$, the maximum correction is only 4%. Thus, even though the velocity correction factors can be as large as 0.5, the thrust correction factors are so low they are likely to be less than the uncertainty of measured wind-tunnel data. This result is consistent with the data measured by Fitzgerald [6] for a family of identical propellers tested in two significantly different-sized wind tunnels. Aside from a few nonrepeatable results (which may be related to the limit conditions identified here), the measured trust levels were essentially the same for a given propeller advance ratio (or power level) to within a small percent. Fitzgerald considered this to be anomalous, but the current results would indicate it is correct. Loeffler and Steinhoff [4] reported similar results from a CFD analysis of a shrouded propeller.

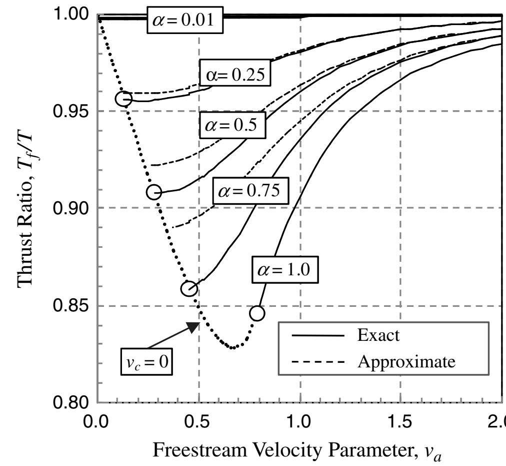

Fig. 5 Thrust correction factor.

Conclusions

Overall, the analytical, experimental, and computational evidence seems to support the assertion that, for nominally small levels of blockage, $\alpha < 0.25$, virtually no correction to the thrust is warranted for wind-tunnel data. For CFD studies, where more precision is possible, faster calculations made with limited sized domains (and thus high blockage levels) can be confidently corrected using Eq. (13b). Additionally, caution needs to be exercised in both experimental and numerical studies to not violate the freestream velocity/power limitation exposed herein.

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